The Pythagoras-Church-Thesis
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What is today known as Church's Thesis ("effective" = computable) was already invoked by the Pythagorean school, dating back at least to the fifth century BC, in the tacit form geometry =arithmetic. This asserted the primacy of number and counting over geometry and spatial extension. For the Pythagoreans, "number" meant "rational number", and this asserted something (commensurability) about spatially extended lengths and how they are measured and / or constructed. Ironically, the Pythagorean Theorem showed this presumption to be false; perfectly effective processes in two dimensions had no one-dimensional arithmetic counterpart. The response was to try to extend arithmetic into infinite realms in order to preserve the thesis. This in turn led to paradoxes, which Zeno was the first to notice. To counter these, still further extensions of arithmetic had to be made, resulting among other things in the fact that most subsets of the line then turn out to have no objective "length" at all (non measurable). We will examine this history, and some of their considerable scientific and conceptual implications.